Level 1 Problem Sets

The topics covered are:

- Natural Numbers (72 questions)
- Integers and Rational Numbers (115 questions)
- Algebraic Expressions (81 questions)
- Algebraic Equations (98 questions)

Each set consists of questions that three levels (basics, intermediate and challenge) of increasing difficulty.

Basics questions cover the essential skills required by student for them to pass their tests and exams.

Being able to work through **Intermediate questions** would indicate that students have a strong foundation, and they would be able to attempt and answer multi-step questions correctly.

Challenge questions are competition type questions that require a strong understanding of mathematics. Many of these questions are word type questions that require a careful interpretation of the given context.

Relevant school level by country

Country	Level
US	Grade 7 - 8
United Kingdom	Year 8 - 9
Australia	Year 7 - 8
New Zealand	Year 8 - 9

Distribution

We will email PDF electronic copies

Pricing

US $30 per topic

NATURAL NUMBERS

BASICS

01 Find the prime factorisation of each of the following numbers.

a 72
b 180
c 250

02 Find the number of factors of each of the following.
a 3^7
b $2^4 \times 5^3$
c $2^5 \times 3^3 \times 7$

03 Find all possibles values of natural numbers n such that $\dfrac{196}{n}$ is a natural number.

04 List all of the prime numbers between 50 and 80.

05 Which of the following is **false**?
a $2 \times 2 \times 2 \times 9 \times 8 = 2^6 \times 3^2$
b $5 \times a \times a \times b = 5 \times a^2 \times b$
c $a \times a \times a \times b \times 4 = 2^2 \times a^3 \times b$
d $x \times x \times y \times x \times z \times z = x^3 \times y \times z^2$
e $\dfrac{1}{x} \times \dfrac{1}{x} \times \dfrac{1}{x} \times \dfrac{1}{y} \times \dfrac{1}{y} = \dfrac{1}{3 \times x \times y^2}$

06 Which of the following pairs of natural numbers are relatively prime?
a 6 and 10
b 17 and 51
c 12 and 33
d 18 and 26
e 21 and 65

07 The Greatest Common Divisor of two numbers is 18.
Find the number of common factors of these two numbers.

INTERMEDIATE

01 Dividing 34 by a natural number n leaves a remainder of 6.
Find all possible values of n.

02 Natural numbers a and b are primes numbers less or than equal to 100 satisfying $a - b = 4$ and $5 < a < 35$.
Find the sum of all possibles values of b.

03 The Greatest Common Divisor and Lowest Common Multiple of $2^m \times 3 \times 7$ and $2^n \times 3^2 \times 5$ are 6 and 1,260, respectively.
Find the value of $m + n$.

04 Dividing a natural number a by 7 gives a qutient of 9 and a remainder that is prime.
Find all possible values of a.

05 The Greatest Common Divisor of two natural numbers is 100.
Find the number of factors common to both the numbers.

06 $n = 2 \times p^2$, where p is a prime number and $p > 2$.
Find the number of factors of n.

07 Identical rectangular tiles that measure 16 cm by 12 cm are put together to form the smallest possible square.
Find the number of tiles required.

08 A rectangle has a width x cm and a length y cm, where x and y are natural numbers.
If the area of the rectangle is 126 cm^2, find the number of distinct rectangles there can be.
Assume that the rectangle remains the same when its width and length are interchanged.

09 When 1 is subtracted from a four-digit number $5\square43$, the resulting number is a multiple of 9.
Find $\square$.

10 Dividing some number by 15 leaves a remainder of 12.
Find remainder when this number is divided by 5.

12 2012 and 2016 are leap years.

13 The GCD of 21 and 63 is 21.

14
The multiples of the Lowest Common Multiple of two numbers are indeed the common multiples.

Therefore, only statement **d** is true.

15 Since $6 \times 166 = 996$ and $6 \times 167 = 1002$, so 1,002 is closest ti 1,000.

16 Since $98 = 2 \times 7^2$, the prime factors are 2 and 7.

17
Statement **a** is false.
If two numbers have only have 1 as the common factor, they are relatively prime.

Statement **d** is also false.
If a and b are relatively prime, where a and b are natural numbers, their Lowest Common Multiple is their product, ab.

All of the other statements are true.

18

Number	Prime factorisation	Number of factors
54	2×3^3	$(1+1) \times (3+1) = 8$
125	5^3	$3+1=4$
108	$2^2 \times 3^3$	$(2+1) \times (3+1) = 12$
210	$2 \times 3 \times 5 \times 7$	$(1+1)(1+1)(1+1)(1+1) = 16$
405	$3^4 \times 5$	$(4+1)(1+1) = 10$

Therefore, 210, 108, 405, 54 and 125.

19
Since 5 2 ■ is a multiple of 3, the possible values are 2, 5 and 8.

Since 7 ■ 2 is a multiple of 2, any nubmer from 0 to 9 is possible.

Therefore, the smallest possible value for ■ is 2.

INTEGERS AND RATIONAL NUMBERS

BASICS

Evaluate each of the following expressions in Questions **01 − 12**.

01 $(-1)^{105} + (-1)^{300} - (-1)^{37}$

02 $(-2) + (-5) \times (-1) \div \left(-\frac{2}{3}\right) - \frac{1}{2}$

03 $\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) + \left(-\frac{5}{6}\right) \times \frac{2}{15}$

04 $\frac{3}{4} \div \left(-\frac{1}{2}\right)^2 - 2^2 \times \frac{7}{4}$

05 $-\frac{2}{3} + \frac{5}{8} \times \left(-\frac{4}{5}\right) + \left(-\frac{1}{2}\right) \div \left(-\frac{3}{4}\right)$

06 $15 - \Big(13 + \big((-4) \times 3 - (4-7) + 10\big)\Big)$

07 $(-18) \div 3 + \Big(6 + (-9)\Big) \times (-2) + (-1) \times (-2)$

08 $\frac{1}{6} \times \left(21.5 - \left(3 + \left(\frac{1}{4} - \frac{1}{6}\right) \times 6\right)\right)$

09 $1 - \left(\frac{1}{2} + (-1) \div \Big(5 \times (-2) + 6\Big)\right)$

10 $7 - 6 \div \left(4 + \left(3 - 10 \times \frac{1}{2}\right)\right) \times (-2) \times \left(-\frac{1}{3}\right)$

11 $2 \times (-1)^3 - \frac{9}{2} \div \left(5 \times \left(-\frac{1}{2}\right) + 1\right)$

12 $\left(\frac{2}{3} - (-1.25)^2 \times 1\frac{3}{5}\right) \div 0.6$

13 Find the number that is -4 less than 5.

14 It is given that $a + 2 = 7$ and $b + (-2) = 7$.
Find the values of a and b.

15 Determine the smallest from the following, where x is a negative number.

$$-3 \times x, \ -5 \times x, \ 3 \times x \text{ and } 5 \times x$$

16 A point that corresponds to the number n is moved 2 units to the right on the number line.
Find an expression for the number that the new point corresponds to on the number line .

17 Which one of the following numbers is the furthest from the origin when represented on the number line?

a $\frac{15}{2}$

b $+6$

c 0

d -2.4

e -8.1

18 Which one of the following statements is **false** regarding the numbers shown below?

-8	$+2\frac{3}{8}$	2.4	$-\frac{9}{3}$	0	$-\frac{5}{4}$	$\frac{8}{2}$

a Three numbers are integers

b The greatest number is $+\frac{8}{2}$

c The smalleast absolute value is that of 0

d There are three rational numbers that are not integers

e $+2\frac{3}{8}$ is the third number when the numbers are arranged in descending order

19 It is given that $(-4)^2 \times a > 0$ and $(-3)^3 \times b > 0$.
Determine the signs of a and b.
Use inequalities in your answers.

INTERMEDIATE

01 a less than 2 is equal to -7 and 8 more than a is equal to b.
Find the values of a and b.

02 Find the smallest and largest integers that are between $-4\frac{2}{3}$ and -0.5.

03 If $|x| = 3$, find all possible values of $\frac{1}{4} - x$.

04 Three numbers are multiplied from the following.

$$-\frac{2}{3} \qquad \frac{7}{2} \qquad 0.8 \qquad -2$$

Find the largest possible number that can be obtained.

05 a is an integer that exceeds -10 and less than -4. Also, a is less than -8.
Find the value of a.

06 Find all integers between $-\frac{29}{7}$ and 1.1.

07 If $a = \frac{1}{4} + \left(-\frac{1}{6}\right)$, find the value of b that satisfies $a \times b = 1$.

08 Integers a and b satisfy $a + b > 0$, $a^2 > 0$ and $a \times b = 0$.
Show that $a > b$.

09 Consider the five numbers shown below.

$$-0.2 \qquad -\frac{1}{4} \qquad 0 \qquad 0.23 \qquad \frac{1}{3}$$

Find the each of the following.
a The number with the greatest absolute value
b The number with the smallest absolute value

CHALLENGE

01 It is given that $|a| > |b|$.
Write down the appropriate number or sign (0, $-$ or $+$) for each of the cases in the table below.

	$a+b$	$a \times b$
$a > 0$ and $b > 0$		
$a > 0$ and $b < 0$		
$a < 0$ and $b > 0$		
$a < 0$ and $b < 0$		
$a > 0$ and $b = 0$		
$a < 0$ and $b = 0$		

02 Integers A and B satisfy $|A| = |B|$ and $A - B = 8$.
Find the values of A and B.

03 The sum of the scores of Mark, Yuri, Esther, Ethan and Jay is 0.

a The scores of Mark, Yuri, Esther and Jay are 15, -8, -4 and 1.
Find Ethan's score.

b The average score of Mark, Yuri, Esther and Ethan is -3.5, find Jay's score.

04 Rational numbers a, b and c satisfy $a \times b > 0$, $a \times b \times c \leq 0$ and $a + b < 0$.
Determine the sign of $a + b + c$.

05 Two numbers from $-1\frac{1}{3}$, 0.25 and $\frac{3}{4}$ are multiplied and the result is divided by the remaining number.

If the absolute value of the quotient must be the maximum possible value, find the quotient.

10

$$\left(-\frac{1}{4}\right) \div \left(-\frac{1}{2}\right)^3 - (-6) \times \left(\frac{3}{4} + (-2)\right) = \left(-\frac{1}{4}\right) \times (-8) + 6 \times \left(\frac{3}{4} - \frac{8}{4}\right)$$
$$= 2 + 6 \times \left(-\frac{5}{4}\right)$$
$$= 2 - \frac{15}{2}$$
$$= -\frac{11}{2}$$

11

$$\left|-2^3 \div 3\right| - \left|-2\frac{1}{3} \div \left(-1\frac{5}{9}\right)\right| = \left|-\frac{8}{3}\right| - \left|-\frac{7}{3} \div \left(-\frac{14}{9}\right)\right|$$
$$= \left|-\frac{8}{3}\right| - \left|-\frac{7}{3} \times \left(-\frac{9}{14}\right)\right|$$
$$= \left|-\frac{8}{3}\right| - \left|\frac{3}{2}\right|$$
$$= \frac{8}{3} - \frac{3}{2}$$
$$= \frac{16-9}{6}$$
$$= \frac{7}{6}$$

12

$$-\left|-\left|\frac{(-3)^2}{-2^2}\right|\right| = -\left|-\left|-\frac{9}{4}\right|\right|$$
$$= -\left|-\frac{9}{4}\right|$$
$$= -\frac{9}{4}$$

13

$$2-\left(\frac{1}{2}+\left(-\frac{2}{3}\right)\times\left(\frac{7}{2}+\left(-\frac{5}{6}\right)\times\frac{8}{5}\right)\right)\div\frac{1}{3}=2-\left(\frac{1}{2}+\left(-\frac{2}{3}\right)\times\left(\frac{7}{2}-\frac{4}{3}\right)\right)\div\frac{1}{3}$$
$$=2-\left(\frac{1}{2}+\left(-\frac{2}{3}\right)\times\frac{13}{6}\right)\div\frac{1}{3}$$
$$=2-\left(\frac{1}{2}-\frac{13}{9}\right)\times 3$$
$$=2+\frac{17}{18}\times 3$$
$$=2+\frac{17}{6}$$
$$=\frac{29}{6}$$

14 Since M is halfway between points A and B, the number that corresponds to point M is $\frac{-10+4}{2}=-3$.

15
$a\times b>0$ means that a and b are of the same sign.

But since $a+b<0$, $a<0$ and $b<0$.

16 $|n|=3+m=13$. But $10\times n<0$ which simplifies to $n<0$, so we can conclude that $n=-13$.

17 When $a=2$ and $b=-5$, $a-b=2-(-5)=7$, which is the maximum value that $a-b$ can take.

18 $[-2.3]=-3$.

19
Since $a+(-3)=a-3$ is a positive integer, $a>3$.

Since $a+(-5)=a-5$ is a negative integer, $a<5$.

Since $3<a<5$, $a=4$.

32 $162 = 2 \times 3^4$, so the numbers are $(3, 6, -9, -1, 1)$ and $(-3, -6, 9, -1, 1)$.

33
$\frac{6}{17}$ is part of the 22th group. e.g. $\left(..., \frac{6}{17}, \frac{5}{18}, \frac{4}{19}, \frac{3}{20}, \frac{2}{21}, \frac{1}{22}\right)$

$(1 + 2 + ... + 22) - 5 = \frac{22 \times 23}{2} - 5 = 253 - 5 = 248$

Therefore, $\frac{6}{17}$ is 248th number.

34 Let + be right from the starting point, and − be left from the starting point.

a
Julia is $3 \times 4 - 2 \times 3 = 6$ steps right from the starting point.

$2 \times 4 - 3 \times 3 = -1$ so Jasmine is 1 step left from the starting point.

Therefore, Julia and Jasmine are $6 - (-1) = 7$ steps apart.

b
Let x be the number of times Jasmine won.

$$\begin{aligned} 4 \times x - 3 \times (7 - x) &= 0 \\ 4x - 21x + 3x &= 0 \\ 7x &= 21 \\ x &= 3 \end{aligned}$$

Therefore, Jasmine won 3 times and lost 4 times.

ALGEBRAIC EXPRESSIONS

BASICS

01 Which of the following is the **correct** simplification of the expression $a \div b \times c \div d$?

a $\frac{ad}{bc}$

b $\frac{bc}{d}$

c $\frac{bc}{ad}$

d $\frac{ac}{bd}$

e $\frac{a}{bcd}$

02 Which of the following are **correct**?

a $a \div b \times c = \frac{a}{bc}$

b $a \div b \times c = a \div (bc)$

c $a \times b \div c = a \div c \times b$

d $a \div b \div c = a \div (b \div c)$

e $a \div b \times c = ac \div b$

03 Which of the following is equal to the expression $\frac{a+b}{xy}$?

a $(a+b) \div x \div y$

b $(a+b) \div x \times y$

c $a + b \div x \div y$

d $a + b \div x \times y$

e $a \div x + b \div y$

04 Which of the following are linear expressions? Choose all correct answers.

a $x^2 - 3x + 1$

b $(3-3)x + 5$

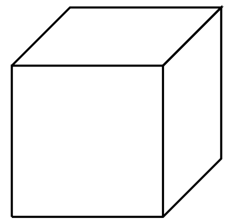
c $2x + 3 - 2x$

d $0.2x - 3$

e $2x^2 - 4x + 3 - x - 2x^2$

12 Express the combined length of a cm and b m in millimetres.

13 A cube has a side length of x.



Find a simplified expression for the surface area of the cube in terms of x.

14 Samuel travels at a km per hour for y minutes.
Find an expression for the distance he travelled in metres.
Express your answer as an improper fraction in terms of a and y.

15 Find a simplified expression for 30% of x litres.

16 Find an expression for the amount of salt, in grams, in 0.3 kg of x% saline solution.

17 The minute hand of an analogue clock turns by $x°$.
Find an expression for the number of degrees that the hour hand turns during this time in terms of x.

18 Find an expression for the number of seconds in x hours and y minutes.

19 Simplify $x + y - \Big(x + y - \big((x - y) - (x + y)\big)\Big)$.

20 Simplify $\dfrac{x-1}{4} + \dfrac{2x-2}{3} - \dfrac{2x+5}{2}$.

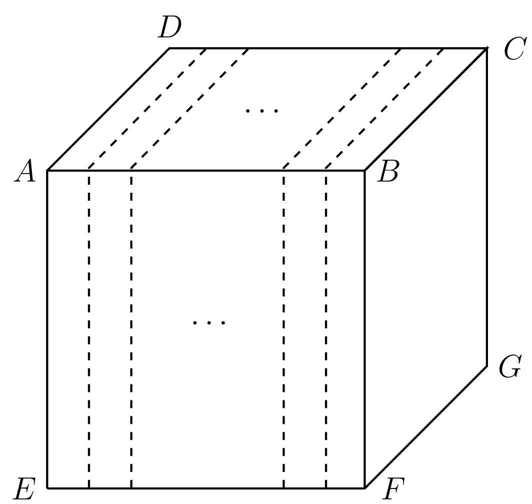
21 Determine the constant term and coefficient of x after simplifying the following expression.

$$15\left(\frac{2}{3}x - \frac{1}{5}\right) - 12\left(\frac{1}{4} - \frac{5}{6}x\right)$$

.

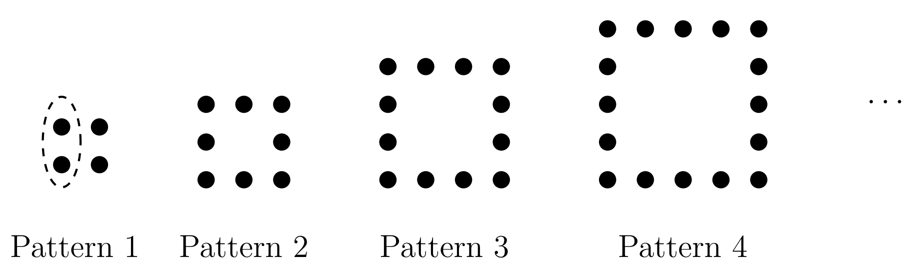
23 Last year, a supermarket chain sold x tonnes of apples and $(x - 30)$ tonnes of oranges.
This year, 2% more apples were sold and 1% fewer oranges were sold.
Express the percentage increase in the combined number of apples and oranges sold from last year in terms of x.

24 A cube has a side length of 2 cm.
The cube is cut n times along a plane parallel to $BFGC$.



Find the total surface area of the resulting slices in terms of n.

25 For each side of the following pattern of shapes, 2 circles, 3 circles, 4 circles and 5 circles are used respectively.



Let c be the number of circles used for each side in a shape. For example, $c = 2$ for Pattern 1.
Answer the following questions.

a Find the number of circles used to form a shape with 6 circles on one side.

b Show that the total number of circles used in a shape is a multiple of 4.

c Consider all shapes where the total number of circles in a shape is a multiple of 8.
Determine the condition on the number c for these shapes.

10

$$\begin{aligned} -3(2a-3b)+5(3a-b) &= -6a+9b+15a-5b \\ &= 9a+4b \end{aligned}$$

The coefficients of a and b are 9 and 4, respectively.

11

$$\begin{aligned} (a-b+3c)^3 &= \Big(4-(-3)+3(-2)\Big)^3 \\ &= (4+3-6)^3 \\ &= 1^3 \\ &= 1 \end{aligned}$$

12
Note that 1 cm = 10 mm and 1 m = 1000 mm.

The combined length is $(a \times 10 + b \times 1000) = (10a + 1000b)$ mm.

13 $6 \times x^2 = 6x^2$

14 Distance = speed × time = $\dfrac{a \times 1000}{60} \times y = \dfrac{50ay}{3}$ m

15 $x \times \dfrac{30}{100} = \dfrac{3x}{10}$ litres

16 $0.3 \times \dfrac{x}{100} \times 1000 = 3x$ g

17
The minute hand turns by 360° when the hour hand turns by 30°.

Therfore, the number of degrees that the hour hand turns is
$\dfrac{30^\circ}{360^\circ} \times x^\circ = \dfrac{x^\circ}{12}$.

18 $x \times 3600 + y \times 60 = (3600x + 60y)$ seconds.

19

$$
\begin{aligned}
x+y-\Big(x+y-\big((x-y)-(x+y)\big)\Big) &= x+y-\Big(x+y-\big(x-y-x-y\big)\Big)\\
&= x+y-\Big(x+y-\big(-2y\big)\Big)\\
&= x+y-(x+y+2y)\\
&= x+y-(x+3y)\\
&= x+y-x-3y\\
&= -2y
\end{aligned}
$$

20

$$
\begin{aligned}
\frac{x-1}{4}+\frac{2x-2}{3}-\frac{2x+5}{2} &= \frac{3(x-1)}{12}+\frac{4(2x-2)}{12}-\frac{6(2x+5)}{12}\\
&= \frac{3(x-1)+4(2x-2)-6(2x+5)}{12}\\
&= \frac{3x-3+8x-8-12x-30}{12}\\
&= \frac{-x-41}{12}\\
&= -\frac{x+41}{12}
\end{aligned}
$$

21

$$
\begin{aligned}
15\Big(\frac{2}{3}x-\frac{1}{5}\Big)-12\Big(\frac{1}{4}-\frac{5}{6}x\Big) &= 10x-3-3+10x\\
&= 20x-6
\end{aligned}
$$

The constant term is -6 and coefficient of x is 20.

22

$a \div b = -2$ can be written as $a = -2b$.

Substituting this equation into $a - b = 12$ gives

$$
\begin{aligned}
(-2b)-b &= 12\\
-3b &= 12\\
b &= -4
\end{aligned}
$$

It follows that $a = -2(-4) = 8$.

21
The first shape has 5 matchsticks and each subsequent shape has 3 more matchsticks.

Therefore, the number of matchsticks for the nth shape is $5 + 3(n-1) = 5 + 3n - 3 = 3n + 2$.

22
The amount of salt in beaker A is $200 \times \dfrac{a}{100} = 2a$ g.

The amount of salt in beaker B is $300 \times \dfrac{b}{100} = 3b$ g.

After pouring 100 g of solution in beaker A (a g of salt) into beaker B, the amount of salt in beaker B is $(a + 3b)$ g.

This means that the amount of salt in 100 g of the mixture is $100 \times \dfrac{a+3b}{400} = \dfrac{a+3b}{4}$ g.

Once 100 g of the mixture is poured into beaker A, its concentration is
$$\frac{a + \dfrac{a+3b}{4}}{200} \times 100 = \frac{5a+3b}{800} \times 100 = \frac{5a+3b}{8}\ \%.$$

23
The combined number of apples and oranges sold this year is
$$x\frac{102}{100} + (x-30)\frac{99}{100} = \frac{102x + 99(x-30)}{100} = \frac{201x - 2970}{100} = 2.01x - 29.7.$$

Therefore, the percentage increase from last year is
$$\begin{aligned}\frac{(2.01x - 29.7) - (2x - 30)}{x + x - 3} \times 100 &= \frac{0.01x + 0.3}{2x - 30} \times 100 \\ &= \frac{x+30}{2x-30}\ \%\end{aligned}$$

24
The surface area of the cube is $6 \times 2^2 = 24$ cm^2.

Every time the cube is cut, the surface area of the solids increases by 8 cm^2 (two squares).

Therefore, the total surface area is given by $24 + 2n \times 4 = (24 + 8n)$ cm^2.

25
a $6 \times 2 + (6-2) \times 2 = 20$

b The number of circles used is given by $2c + (c-2) \times 2 = 4c - 4 = 4(c-1)$.

c
We require $4(c-1) = 8k$, where k is a natural number. This can be written as $c = 2k + 1$.
Therefore, c must be an odd number.

ALGEBRAIC EQUATIONS

BASICS

01 Which of the following are linear equations?

a $2x - 4 = -2(2 - x)$

b $x - 6 = x$

c $x + x = 2x$

d $2x^2 + 4 = 2x^2 + x + 1$

e $2(x^2 + x + 1) = 2x^2 + 4x + 1$

02 Which of the following are **not** identities?

a $5x + 4x = 9x$

b $3x + 5 = 7x + 8$

c $3x + 15 = 3(x + 5)$

d $10x + 2 = 2 + 10x$

e $x^2 + x + 1 = x^2 + 2x + 1 + x$

03 Which of the following properties of equations are used in solving the equation $2x + 10 = 20$?

If $A = B$ and $C > 0$, then	
I. $A + C = B + C$	II. $A - C = B - C$
III. $AC = BC$	IV. $A \div C = B \div C$

a I and III

b II and III

c II and IV

d III and IV

e I, II, III and IV

04 Solve each of the following questions.

a $2x - 5 = 3$

b $-x + 4 = 2$

c $\frac{1}{2}x - 3 = -1$

d $0.5x + 7 = 9$

e $2x + 1 = 3x - 3$

INTERMEDIATE

01 Consider the following equations, where p and q are constants.

$$\begin{array}{ll} 3(x-5) = 4(2x-3) - 8 & \cdots (1) \\ p(x+1) + 2(q-1) - 3 = 0 & \cdots (2) \end{array}$$

The value of x that solves equation (2) is three times the value of x that solves equation (1).
Find the value of $2p + q$.

02 The equations $(2x+1) : (3x-1) = 3 : 4$ and $(2x+a) : (3x-a) = 4 : 3$ have the same solution, where a is a constant.
Find the value of a.

03 It is given that $x = \dfrac{4a-b}{a+b}$ is the solution to the equation $-3x + m = -1$, where m is a constant.

Additionally, $a - b = 2a - 3b$ and $a + b \neq 0$.

Find the value of m.

04 Let $\min(a, b)$ be defined as the smallest number of distinct numbers a and b. For example, $\min(7, 3) = 3$.
Solve each of the following equations.

a $\dfrac{\min(6,9)}{2} = \min(5, 9-3x)$

b $\min(x-1, 3) = 2x$

05 Consider the equation $ax - 3 = 2x + b$, where a and b are constants.
Determine the values of a and b for each of the following cases.

a The equation has infinitely many solutions.

b The equation has no solutions.

06 Let $\max(a, b)$ be defined as the largest number of a and b. For example, $\max(3, 5) = 5$.
Solve the equation $\max(x-2, 3) + \max(5-x, 1) = 7$ for each of the following cases.

a $x < 4$

b $4 < x < 5$

c $x > 5$

CHALLENGE

01 Solve $\left|3x + \left|x - 3\right|\right| = 5$.

02 Solve $x + 1 = \left|x\right| + \left|x - 3\right|$.

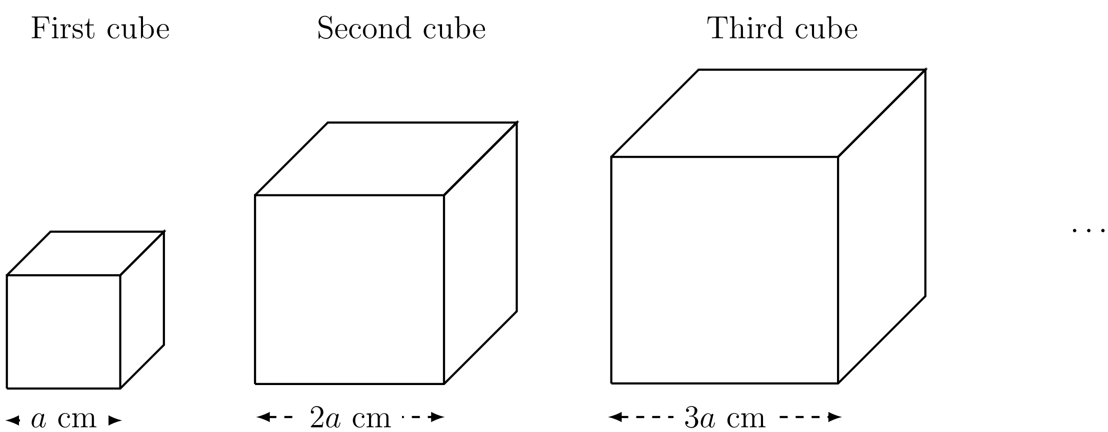
03 For any two numbers a and b, let $\langle a, b\rangle = ax + b$, where x is a real number.

a Find the missing letters and number in each of $\square$ below.

$\langle a, b\rangle + \langle c, d\rangle = \langle \square, \square\rangle$
$a\langle c, d\rangle = \langle \square, \square\rangle$
If $\langle a, b\rangle = \langle c, d\rangle$ for all values of x, it follows that $a = \square$ and $b = \square$

b If $\langle 3, -7\rangle = -1$, show that $\langle 1, 0\rangle = 2$.
c If $2\langle 1, 0\rangle = \langle 0, 11\rangle - \langle -1, 1\rangle$, evaluate $\langle 1, 0\rangle$.

04 Consider a pattern of cubes. Each cube has a number displayed on each face.
The first cube with a side length of a cm is numbered 1 to 6.
The second cube with a side length of $2a$ cm is numbered 7 to 12.
The third cube with a side length of $3a$ cm is numbered 13 to 18.



a Find the sum of the numbers displayed on the cube with a side length of na cm, where n is an integer.

b The sum of the numbers on one of the cubes in the pattern is 597.
Find the length of this cube in terms of a.

04

case 1 when $5 < 9 - 3x$ or $x < \dfrac{4}{3}$

$$\begin{aligned}\frac{\langle\langle 6,9\rangle\rangle}{2} &= \langle\langle 5, 9-3x\rangle\rangle \\ \frac{6}{2} &= 5 \\ 3 &= 5\end{aligned}$$

But this is a contradiction so this means that there are no solutions in this case.

case 2 when $5 > 9 - 3x$ or $x > \dfrac{4}{3}$

$$\begin{aligned}\frac{\langle\langle 6,9\rangle\rangle}{2} &= \langle\langle 5, 9-3x\rangle\rangle \\ 3 &= 9-3x \\ 3x &= 6 \\ x &= 2\end{aligned}$$

This is consistent with the restriction $x > \dfrac{4}{3}$. Therefore, the solution to the equation is $x = 2$.

b

case 1 when $x - 1 < 3$ or $x < 4$

$$\begin{aligned}\langle\langle x-1, 3\rangle\rangle &= 2x \\ x-1 &= 2x \\ -1 &= x\end{aligned}$$

case 2 when $x - 1 > 3$ or $x > 4$

$$\begin{aligned}\langle\langle x-1, 3\rangle\rangle &= 2x \\ 3 &= 2x \\ \frac{3}{2} &= x\end{aligned}$$

But since $x = \dfrac{3}{2}$ does not satisfy $x > 4$, it cannot be a solution.

Therefore, the solution to the equation is $x = -1$.

05

$$\begin{aligned}ax - 3 &= 2x + b \\ ax - 2x &= 3 + b \\ (a-2)x &= 3 + b\end{aligned}$$

a The equation has infinitely many solutions when $a = 2$ and $b = -3$.

b The equation has no solutions when $a = 2$ and $b \neq -3$.

27

a $\dfrac{x-10}{12}$ hours.

b

$$\begin{aligned}\frac{x-10}{10}+\frac{10}{60}&=\frac{x-10}{12}+\frac{50}{60}\\60\times\left(\frac{x-10}{10}+\frac{10}{60}\right)&=60\times\left(\frac{x-10}{12}+\frac{50}{60}\right)\\6(x-10)+10&=5(x-10)+50\\6x-60+10&=5x-50+50\\x&=50\end{aligned}$$

28

Let s and f be the ages of Sarah and her father.

The given information can be written as the equations $4s+40f=2304$ and $2(s+3)=f+3$.

Substituting $f=2s+3$ into $4s+40f=2304$ gives

$$\begin{aligned}4s+40(2s+3)&=2304\\4s+80s+120&=2304\\84s&=2184\\s&=26\end{aligned}$$

Thus $f=2\times26+3=55$.

29

Let x be the number of days that fell on the weekend when the students use Court A.

	Weekend	Weekday
Court A	x	$8-x$
Court B	$6-x$	$4+x$

$$\begin{aligned}600x+400(8-x)+400(6-x)+300(4+x)&=7000\\600x+3200-400x+2400-400x+1200+300x&=7000\\6800+100x&=7000\\100x&=200\\x&=2\end{aligned}$$

30

Let x be the number of minutes past 5 o'clock when Mimi arrived at the library.

$$\begin{aligned} 6x &= 150 + 0.5x \\ 5.5x &= 150 \\ x &= \frac{150}{5.5} \\ &= \frac{1500}{55} \\ &= \frac{300}{11} \end{aligned}$$

Let y be the number of minutes past 9 o'clock when Mimi left the library.

$$\begin{aligned} (270 + 0.5y) - 6y &= 180 \\ -5.5y &= -90 \\ y &= \frac{90}{5.5} \\ &= \frac{900}{55} \\ &= \frac{180}{11} \end{aligned}$$

$$x - y = \frac{180}{11} - \frac{300}{11} = -\frac{120}{11}$$

$$60 - \frac{120}{11} = \frac{540}{11} = 49\frac{1}{11}$$

Therefore, the duration of Mimi's study is 3 hours and $49\frac{1}{11}$ minutes.